Today you will:

- Visualize solutions to systems of linear equations in three variables
- Solve systems of linear equations in three variables algebraically
- Solve real-life problems
- Practice using English to describe math processes and equations

Examples of Systems of Equations in Two Variables:

$y = -2x - 9$	Equation 1
$6x - 5y = -19$	Equation 2

$-x + y = 3$	Equation 1
$3x + y = -1$	Equation 2

… in Three Variables:

$3x + 4y - 8z = -3$	Equation 1
$x + y + 5z = -12$	Equation 2
$4x - 2y + z = 10$	Equation 3

What does it mean to solve Systems of Equations in Three Variables:

An equation with 2 variables represents a line …

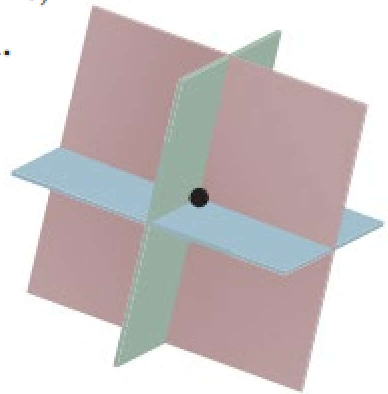
…an equation with 3 variables represents a plane …

So, each of the three equations represents a plane…

Solving the system means finding if and where these three planes intersect

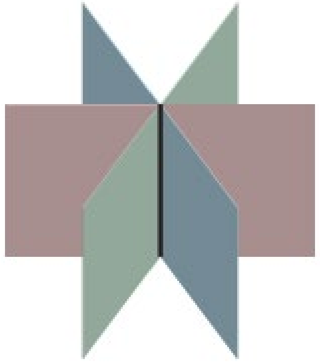
…in other words you will get an x, a y and a z … (x, y, z)

Exactly One Solution
The planes intersect in a single point, which is the solution of the system.

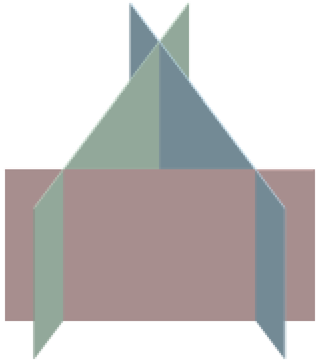


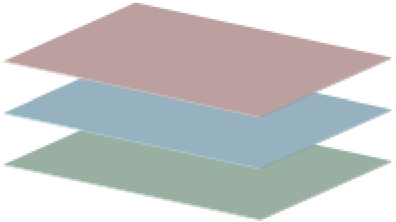
Infinitely Many Solutions
The planes intersect in a line. Every point on the line is a solution of the system.

The planes could also be the same plane. Every point in the plane is a solution of the system.



No Solution
There are no points in common with all three planes.





Solving Systems of Equations in Three Variables Algebraically

Brainstorm with your table group …

… can you think of a way to use what we learned yesterday to help us get going?

Figure out how to eliminate the *y*’s…

$3x + 4y - 8z = -3$	Equation 1	
$x + y + 5z = -12$	Equation 2	
$4x - 2y + z = 10$	Equation 3	

Multiply equation 2 by -4 and add to equation 1

Multiply equation 2 by 2 and add to equation 3

Can you use elimination to “pull” 2 two-variable equations out of these?

…i.e. can you combine two at a time of the above to cancel/eliminate a variable leaving 2 two-variable equations?

…the *y*’s look pretty easy to work with…

Process for solving a System of Linear Equations in three variables algebraically:

1. Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

2. Solve the new linear system (in two variables) for both of its variables.

3. Substitute the values found in step 2 into one of the original equations and solve for the remaining variable.

4. Double check your answers by plugin back into all three equations!

When you do not obtain a false solution, but obtain an identity such as 0 = 0, the system has infinitely many solutions.

LOOKING FOR STRUCTURE

The coefficient of −1 in Equation 3 makes y a convenient variable to eliminate.

ANOTHER WAY

In Step 1, you could also eliminate x to get two equations in y and z, or you could eliminate z to get two equations in x and y.

Solve the system.

$4x + 2y + 3z = 12$ Equation 1

$2x - 3y + 5z = -7$ Equation 2

$6x - y + 4z = -3$ Equation 3

SOLUTION

Step 1 Rewrite the system as a linear system in *two* variables.

$$\begin{array}{rrrr} 4x & +\ 2y & +\ 3z & = 12 \\ 12x & -\ 2y & +\ 8z & = -6 \\ \hline 16x & & +\ 11z & = 6 \end{array}$$

Add 2 times Equation 3 to Equation 1 (to eliminate y).

New Equation 1

$$\begin{array}{rrrr} 2x & -\ 3y & +\ 5z & = -7 \\ -18x & +\ 3y & -\ 12z & = 9 \\ \hline -16x & & -\ 7z & = 2 \end{array}$$

Add −3 times Equation 3 to Equation 2 (to eliminate y).

New Equation 2

Step 2 Solve the new linear system for both of its variables.

$$\begin{array}{rrr} 16x & +\ 11z & = 6 \\ -16x & -\ 7z & = 2 \\ \hline & 4z & = 8 \end{array}$$

Add new Equation 1 and new Equation 2.

$z = 2$ Solve for z.

$x = -1$ Substitute into new Equation 1 or 2 to find x.

Step 3 Substitute $x = -1$ and $z = 2$ into an original equation and solve for y.

$$6x - y + 4z = -3 \qquad \text{Write original Equation 3.}$$

$$6(-1) - y + 4(2) = -3 \qquad \text{Substitute } -1 \text{ for } x \text{ and } 2 \text{ for } z.$$

$$y = 5 \qquad \text{Solve for } y.$$

The solution is $x = -1$, $y = 5$, and $z = 2$, or the ordered triple $(-1, 5, 2)$. Check this solution in each of the original equations.

Solve the system. $x + y + z = 2$ Equation 1

$5x + 5y + 5z = 3$ Equation 2

$4x + y - 3z = -6$ Equation 3

SOLUTION

Step 1 Rewrite the system as a linear system in *two* variables.

$$\begin{array}{r} -5x - 5y - 5z = -10 \\ \underline{5x + 5y + 5z = 3} \\ 0 = -7 \end{array}$$

Add –5 times Equation 1 to Equation 2.

Because you obtain a false equation, the original system has no solution.

ANOTHER WAY

Subtracting Equation 2 from Equation 1 gives $z = 0$. After substituting 0 for z in each equation, you can see that each is equivalent to $y = x + 3$.

Solve the system.

$$x - y + z = -3 \quad \text{Equation 1}$$
$$x - y - z = -3 \quad \text{Equation 2}$$
$$5x - 5y + z = -15 \quad \text{Equation 3}$$

SOLUTION

Step 1 Rewrite the system as a linear system in *two* variables.

$$\begin{array}{r} x - y + z = -3 \\ x - y - z = -3 \\ \hline 2x - 2y \quad = -6 \end{array}$$

Add Equation 1 to Equation 2 (to eliminate z).

New Equation 2

$$\begin{array}{r} x - y - z = -3 \\ 5x - 5y + z = -15 \\ \hline 6x - 6y \quad = -18 \end{array}$$

Add Equation 2 to Equation 3 (to eliminate z).

New Equation 3

Step 2 Solve the new linear system for both of its variables.

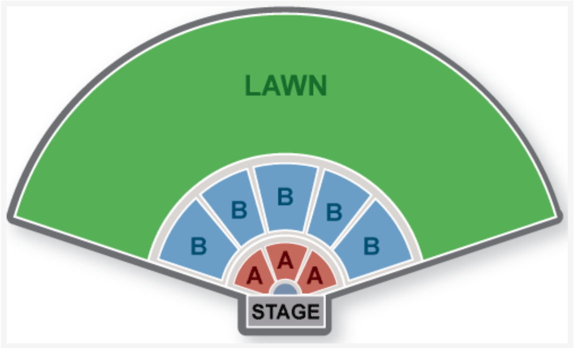
$$\begin{array}{r} -6x + 6y = 18 \\ 6x - 6y = -18 \\ \hline 0 = 0 \end{array}$$

Add -3 times new Equation 2 to new Equation 3.

Because you obtain the identity $0 = 0$, the system has infinitely many solutions.

Step 3 Describe the solutions of the system using an ordered triple. One way to do this is to solve new Equation 2 for y to obtain $y = x + 3$. Then substitute $x + 3$ for y in original Equation 1 to obtain $z = 0$.

So, any ordered triple of the form $(x, x + 3, 0)$ is a solution of the system.



An amphitheater charges \$75 for each seat in Section A, \$55 for each seat in Section B, and \$30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue from selling all 23,000 seats is \$870,000. How many seats are in each section of the amphitheater?

SOLUTION

Step 1 Write a verbal model for the situation.

Number of seats in B, *y* = 3 • Number of seats in A, *x*

Number of seats in A, *x* + Number of seats in B, *y* + Number of lawn seats, *z* = Total number of seats

75 • Number of seats in A, *x* + 55 • Number of seats in B, *y* + 30 • Number of lawn seats, *z* = Total revenue

Step 2 Write a system of equations.

$y = 3x$ Equation 1

$x + y + z = 23{,}000$ Equation 2

$75x + 55y + 30z = 870{,}000$ Equation 3

Step 3 Rewrite the system in Step 2 as a linear system in *two* variables by substituting $3x$ for y in Equations 2 and 3.

$$x + y + z = 23{,}000 \quad \text{Write Equation 2.}$$

$$x + 3x + z = 23{,}000 \quad \text{Substitute } 3x \text{ for } y.$$

$$4x + z = 23{,}000 \quad \text{New Equation 2}$$

$$75x + 55y + 30z = 870{,}000 \quad \text{Write Equation 3.}$$

$$75x + 55(3x) + 30z = 870{,}000 \quad \text{Substitute } 3x \text{ for } y.$$

$$240x + 30z = 870{,}000 \quad \text{New Equation 3}$$

Step 4 Solve the new linear system for both of its variables.

STUDY TIP

When substituting to find values of other variables, choose original or new equations that are easiest to use.

$$\begin{aligned} -120x - 30z &= -690{,}000 \\ \underline{240x + 30z} &\underline{= 870{,}000} \\ 120x \qquad &= 180{,}000 \end{aligned} \quad \text{Add } -30 \text{ times new Equation 2 to new Equation 3.}$$

$$x = 1500 \quad \text{Solve for } x.$$

$$y = 4500 \quad \text{Substitute into Equation 1 to find } y.$$

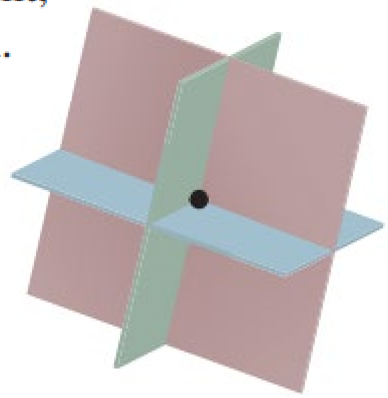
$$z = 17{,}000 \quad \text{Substitute into Equation 2 to find } z.$$

The solution is $x = 1500$, $y = 4500$, and $z = 17{,}000$, or (1500, 4500, 17,000). So, there are 1500 seats in Section A, 4500 seats in Section B, and 17,000 lawn seats.

*** You end up with values for x, y and z ***

Exactly One Solution

The planes intersect in a single point, which is the solution of the system.

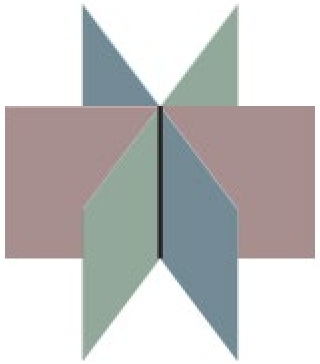


*** You end up with the identity 0 = 0 ***

Infinitely Many Solutions

The planes intersect in a line. Every point on the line is a solution of the system.

The planes could also be the same plane. Every point in the plane is a solution of the system.



No Solution *** You end up with an invalid identity such as 7 = 0 ***

There are no points in common with all three planes.